L_∞ stable if $\|\varepsilon C(t)\| \le \varepsilon c < 1$, which is true, and

$$\varepsilon c + 1/|\xi - c_1| \sup_{t \ge 0} \Lambda\{D(t) + C'(t) - BC(t)\} < 1 \qquad (34)$$

which is true if $|\xi - c_1| > 0(\varepsilon)$.

Remark. This corollary, too, may be generalized to systems governed by the more general equation

$$[A + \varepsilon C(t)]x'(t) = [B + \varepsilon D(t)]x(t) + u(t) \qquad (35)$$

This section now closes with two proposed extensions to this work:

1) Determination of a closed-form expression for $\mathscr{L}^{-1}\{(s^2A + sB + C)^{-1}\}$ would prove extremely useful, as it would enable the second-order system to be analyzed as such.

2) The analysis was done only for systems with the stable constant parts. Stable systems with unstable constant parts form a class not yet explored.

Conclusion

The authors have been recently engaged in the study of the stability of time-varying systems. In continuation with the earlier work, L_∞ stability is studied in this Note. A lemma dealing with the L_∞ stability of an integral equation, resulting from the differential equation of the system under consideration, is first proved. Using it, the main result on L_∞ stability is derived, according to which the system is L_∞ stable, if the eigenvalues of the coefficient matrices are related in a simple way. A corollary of the theorem deals with constant coefficient systems perturbed by small periodic terms, a problem of great importance in its own right. Although the mathematical analysis may seem a little abstract, and may at first deter a practicing engineer not trained in the methods of modern mathematics from using them, the final results are easy to verify even for complex systems.

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Nutation Damping Using a Pivotable Momentum Wheel

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Introduction

A PIVOTABLE momentum wheel can be used as an effective active nutation damping device on momentum-biased spacecraft. Previously described pivot-based active nutation control systems provide linear, essentially viscous damping.[1–3] Although linear damping is effective, it can be slow, requiring many nutation cycles to damp a large initial nutation. The current control system provides much faster damping by driving the pivot with a stepper motor controlled by simple nonlinear digital logic. Simulations and flight data show that nutation angles of 1 deg or more can be damped within two nutation cycles.

System Description

In the following discussion, the spacecraft's yaw, roll, and pitch axes are denoted as X, Y, and Z, respectively. The momentum wheel's spin axis is nominally aligned with the pitch axis. The wheel speed is biased to maintain gyroscopic stiffness and modulated to control pitch. A single-axis pivot mechanism can rotate the wheel in either direction about the roll axis.

Figure 1 outlines the Pivot-Actuated Nutation Damper (PANDA) digital logic. The PANDA pivots the wheel about the roll axis in response to a roll-rate reference. This rate may be obtained directly from a gyro or derived indirectly from an external reference, such as an Earth sensor. In either case, the rate signal is processed by a bandpass filter that passes nutation frequency while removing any biases, structural frequencies, and noise. The filtered roll-rate signal is monitored until it exceeds a predetermined threshold. When the threshold is crossed, the logic determines the sign at the crossing and then determines the amplitude of the sinusoidal rate signal. This amplitude is proportional to the nutation angle. When the filtered roll rate next crosses zero, the momentum wheel pivots through an angle proportional to the roll-rate amplitude. If the zero crossing is from negative to positive, the wheel pivots about the positive roll axis. If the crossing is from positive to negative, the wheel pivots about the negative roll axis. (These signs assume that the spacecraft is dual-spin stable. If the system is dynamically unstable, the sign of the pivoting should be reversed.) One-half nutation period after the first pivoting begins, the wheel pivots an equal angle in the opposite direction. Thus, at the end of the nutation damping cycle, the wheel returns to its initial position. Once the cycle completes, the PANDA logic resumes monitoring the roll rate. For greatest effectiveness, the wheel pivoting time should be short compared to the nutation period. Simulations suggest that a good upper limit for the wheel pivoting time (in one direction) is between 5 and 10% of the nutation period.

Figure 2 illustrates the PANDA dynamics. This momentum diagram shows the trajectory of the tip of the spacecraft's angular momentum vector projected on the body-fixed yaw-roll (X-Y) plane. For simplicity, the figure assumes that the spacecraft's yaw and roll inertias are equal.

In Fig. 2, the momentum wheel is nominally aligned with the positive pitch (Z) axis. If there is no nutation, the nominal trajectory is simply equilibrium point a. When the spacecraft nutates, however, the momentum vector follows a circular

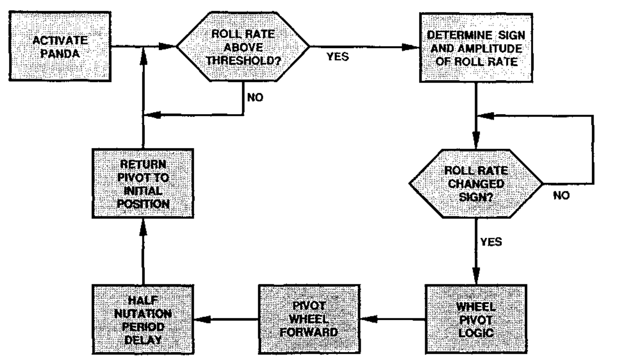

Fig. 1 Simplified pivot-actuated nutation damper logic.

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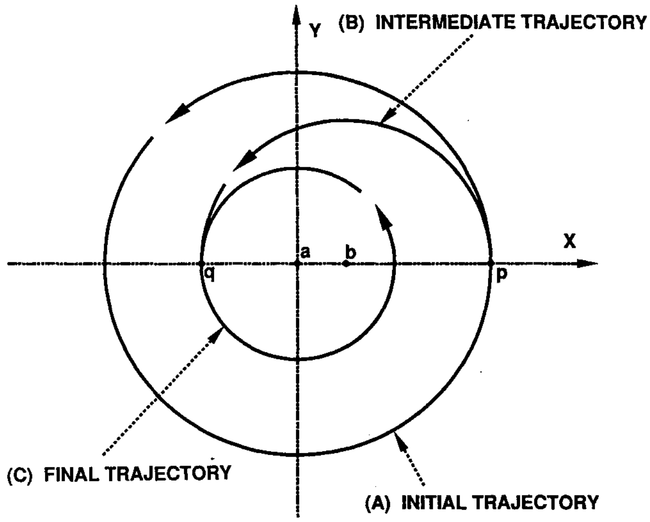

Fig. 2 Angular momentum history due to wheel pivoting during one nutation damping cycle.

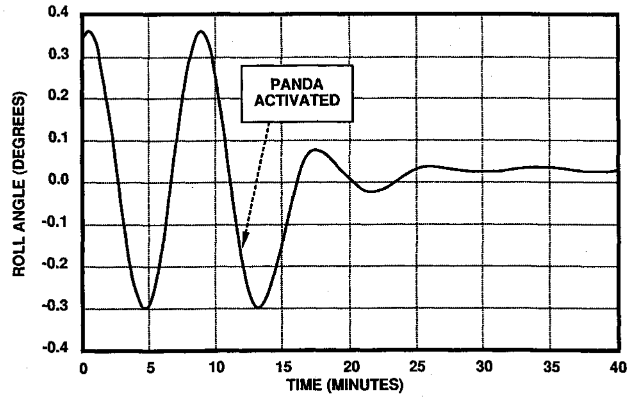

Fig. 3 Roll angle vs time.

path, A, centered on a. The motion along A will be counterclockwise if the spacecraft is dynamically stable. At point p, where the roll rate crosses from negative to positive, the PANDA pivots the momentum wheel about $+Y$. This causes the equilibrium point to move from a to b, and the momentum vector will now follow the circular trajectory B, which is centered at b. Half a nutation period later, when the momentum vector reaches point q, the momentum wheel pivots back to its original position. The momentum vector again circulates around point a, but the new circular trajectory, C, has a smaller radius than the original trajectory, A.

If the time required to pivot the wheel through its commanded angle is a small fraction of the nutation period, then the nutation reduction during one damping cycle is approximately twice the angle between the momentum vector orientations represented by points a and b in Fig. 2. Denoting this angle by ϕ, it can be shown that (for small angles)

$$\phi = \theta h/[h + (I_z - I_x)\omega]$$

where θ is the wheel pivot angle, h the wheel's angular momentum, I_z the pitch moment of inertia, I_x the yaw moment of inertia, and ω the spacecraft's pitch rate. During one damping cycle, the nutation angle is reduced by 2ϕ. If all of the angular momentum is in the wheel (i.e., if $\omega = 0$), then $\phi = \theta$, and the nutation angle is reduced by twice the pivot angle.

Performance

Figures 3 and 4 show typical flight performance data. Here, the PANDA input is a pseudo roll rate derived from the integrated roll gyro output. Figure 3 shows the integrated gyro output (roll angle) vs time. The initial nutation angle is approximately 0.3 deg. Because Fig. 3 plots roll angle, not roll rate, the peak roll rates occur near the zero crossings, and the roll rate is zero at the plotted peaks.

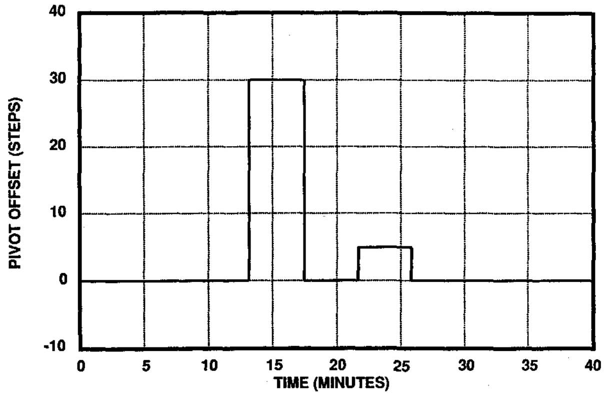

Fig. 4 Pivot offset vs time.

Figure 4 shows the pivot offset (in steps) vs time. On this spacecraft, the stepper motor pivots the wheel 0.00454 deg/step, with the steps occurring at 7.81 Hz. Because some nutation remained after the first damping cycle, a second cycle was required. The second damping cycle reduced the nutation below the 0.01 deg threshold.

The PANDA control logic was nominally designed to damp initial nutation of up to 1 deg or less within one nutation cycle. The flight data show, however, that two cycles were required. The second cycle was required because the PANDA was activated just after the peak roll rate, and, hence, the maximum roll rate detected by the PANDA logic was less than the actual sinusoidal amplitude. The control law commanded a wheel pivot angle proportional to this smaller rate, and only part of the nutation was damped in the first cycle. Had the PANDA been activated a minute earlier, the nutation would have been suppressed completely in the first cycle. Nevertheless, this performance confirms preflight simulations, which showed that no more than two damping cycles are required to suppress nutation that is initially 1 deg or less. These simulations also showed that if the initial nutation exceeds 1 deg the PANDA will remove approximately 1 deg/nutation cycle.

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Stability Analysis of Electro-Magnetoplasmadynamics

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Nomenclature

C_v = constant volume specific heat
C_p = constant pressure specific heat

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